

Warm Up: Solve the system of equations.

a) $x - y = -4 \rightarrow x = y - 4$
 $x + 2y = 5$

$$\begin{aligned} y - 4 + 2y &= 5 \\ 3y &= 9 \\ y &= 3 \\ x &= 3 - 4 = -1 \end{aligned}$$

$(-1, 3)$

b) $6x - 3y - 4 = 0$
 $x + 2y - 4 = 0$

$$\begin{aligned} x &= -2y + 4 \\ 6(-2y + 4) - 3y - 4 &= 0 \\ -12y + 24 - 3y - 4 &= 0 \\ 20 &= 15y \\ y &= \frac{20}{15} = \frac{4}{3} \\ x &= -2\left(\frac{4}{3}\right) + 4 \\ &= -\frac{8}{3} + \frac{12}{3} = \frac{4}{3} \end{aligned}$$

$\left(\frac{4}{3}, \frac{4}{3}\right)$

Examples

Two Solution Case

a) $x - y = -4 \rightarrow x = y - 4$
 $x^2 - y = -2$

$$\begin{aligned} (y - 4)^2 - y &= -2 \\ y^2 - 8y + 16 - y + 2 &= 0 \\ y^2 - 9y + 18 &= 0 \\ (y - 6)(y - 3) &= 0 \end{aligned}$$

$$\begin{aligned} y &= 6 & y &= 3 \\ x &= 6 - 4 = 2 & x &= 3 - 4 = -1 \end{aligned}$$

$(2, 6)$ $(-1, 3)$



b) $x^2 + y = 0 \rightarrow y = -x^2$
 $x^2 - 4x - y = 0$

$$\begin{aligned} x^2 - 4x + (-x^2) &= 0 \\ 2x^2 - 4x &= 0 \\ 2x(x - 2) &= 0 \end{aligned}$$

$$\begin{aligned} x &= 0 & x &= 2 \\ y &= 0 & y &= -(2)^2 = -4 \end{aligned}$$

$(0, 0)$ $(2, -4)$



No Real Solution Case

a) $-\frac{2}{3}x + y = 2$
 $2x - 3y = 6$

$$2x - 3\left(\frac{2x}{3} + 2\right) = 6$$

$$2x - 2x - 6 = 6$$

$$-6 = 6 \quad \times$$

no solution!

b) $x + y = 4$
 $x^2 + y = 2$

$$x^2 - x + 4 = 2$$

$$x^2 - x + 2 = 0 \quad a=1, b=-1, c=2$$

not factorable!

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(2)}}{2(1)}$$

$$= \frac{1 \pm \sqrt{1-8}}{2} = \frac{1 \pm \sqrt{-7}}{2}$$

no real solution!

Applications

A total of \$20,000 is invested in two funds paying 6.5% and 8.5% simple interest. The investor wants a yearly interest check of \$1600 from the two investments. Write and solve a system of equations to determine how much is invested at each interest rate.

x - amount invested @ 6.5% $\rightarrow 0.065$
 y - " " " 8.5% $\rightarrow 0.085$

$$x + y = 20,000 \quad \rightarrow \quad x = 20,000 - y$$

$$0.065x + 0.085y = 1600$$

$$0.065(20,000 - y) + 0.085y = 1600$$

$$1300 - 0.065y + 0.085y = 1600$$

$$0.02y = 300$$

$$y = \$15,000 @ 8.5\%$$

$$x = \$5,000 @ 6.5\%$$

A small fast-food restaurant invests \$5,000 to produce a new food item that will sell for \$3.49. Each item can be produced for \$2.16.

- a) How many units must be sold to break even?
 b) How many units must be sold to make a profit of \$8500?

x - # of items

$$\text{Revenue } (R) = 3.49x$$

$$\text{Cost } (C) = 5,000 + 2.16x$$

$$\textcircled{a} \quad 3.49x = 5000 + 2.16x$$

$$1.33x = 5000$$

$$x \approx 3760 \text{ units}$$

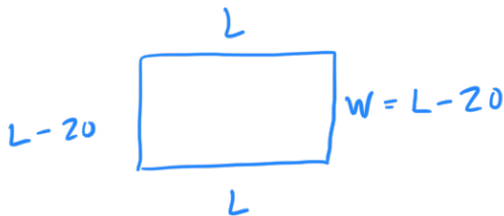
$$\textcircled{b} \quad P = R - C \Rightarrow 3.49x - (5000 + 2.16x) \Rightarrow 1.33x - 5000$$

$$8500 = 1.33x - 5000$$

$$13,500 = 1.33x$$

$$x \approx 10,151 \text{ units}$$

The perimeter of a rectangle is 280 cm and the width is 20 cm less than the length. Find the dimensions of the rectangle.



$$2L + 2(L-20) = 280$$

$$2L + 2L - 40 = 280$$

$$4L = 320$$

$$L = 80 \text{ cm}$$

$$W = 80 - 20 = 60 \text{ cm}$$

Homework: p. 503, #7, 10, 23, 35, 49, 63
 p. 516, #49